International Journal of Engineering Research
Volume No.6, Issue No.1, pp : 38-43

ISSN:2319-6890 (online),2347-5013(print)
1 Jan. 2017

Resolution of Differentials Equations of Excess Minority Carriers and Excitons Transport in the Silicon Base

Mamadou NIANE*, Saliou NDIAYE, Modou FAYE, Modou PILOR, Ousmane NGOM,Nacire MBENGUE,Moulaye DIAGNE, Omar A. NIASSE, Bassirou BA
Laboratoire de Semi-conducteurs et d'Energie Solaire, Département de Physique, Faculté des Sciences et Techniques (UCAD-SENEGAL)
futuringe@gmail.com

***Abstract:** In this work, we proposed a solving method of differential equations transport governing excess minority carriers and excitons density in illuminated silicon base solar cell. To do this, we have transformed this equations system in a linear differential equation with one unknown (excess minority carriers density) that we applied the Laplace transform method. We obtain thus the Laplace transform expression of excess minority carriers density in the base. For applying a reverse Laplace transform on the latter, we obtain the expression of excess minority carriers density in function to base parameters but above in function to two new parameters such as the second and third derivate of excess electrons density at the junction. The expression of excess excitons density is deduced in easy way to the transport equation carriers.*

Keywords: Laplace transform, excess minority carriers, exciton, binding coefficient

INTRODUCTION

The transport equations of excess minority carriers and excitons in a silicon solar cell base are governed in accordance with some assumptions by a system of differential equations [1]. The resolution of this system used to determine the expression of the excess minority carriers density in the base and that of excitons which are essential for determination of the current density produced in the solar cell. A solving method for this system was proposed by Richcard and al [1]. As this differential equations system is linear, we suggested in this article a solving method for this system by a Laplace transform.

I. THEORY

A. DETERMINATION OF ELECTRON DENSITY

The differential equation system described below is:

$$D_e \frac{d^2\Delta n_e}{dx^2} = \frac{\Delta n_e}{\tau_e} + b\left(\Delta n_e N_A - \Delta n_x n^*\right) - G_{oe}\exp(-\alpha x) \quad (1)$$

$$D_x \frac{d^2\Delta n_x}{dx^2} = \frac{\Delta n_x}{\tau_x} - b\left(\Delta n_e N_A - \Delta n_x n^*\right) - G_{ox}\exp(-\alpha x) \quad (2)$$

We must consolidate the differential equations system into a single differential equation with one unknown. For do it, we will express the exciton density from the differential equation (1) in function to the excess minority carriers density, weobtain the following expression:

$$\Delta n_x = \frac{1}{bn^*}\left(\frac{1}{\tau_e} + bN_A\right)\Delta n_e - \frac{D_e}{bn^*}\frac{d^2\Delta n_e}{dx^2} - \frac{G_{oe}}{bn^*}\exp(-\alpha x) \quad (3)$$

By replacing Δn_x for this expression in the equation (2), we obtain the following differential equation:

$$\left[\frac{D_e D_x}{bn^*}\right]\frac{d^4\Delta n_e}{dx^4} - \left[\frac{D_e}{bn^*}\left(bn^* + \frac{1}{\tau_x}\right) + \frac{D_x}{bn^*}\left(bN_A + \frac{1}{\tau_e}\right)\right]\frac{d^2\Delta n_e}{dx^2}$$
$$+\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*} + \frac{1}{bn^*\tau_e}\right) + \frac{1}{\tau_e}\right]\Delta n_e = \left[G_{ox} + G_{oe} + \frac{G_{oe}}{bn^*}\left(\frac{1}{\tau_x} - D_x\alpha^2\right)\right]\exp(-\alpha x) \quad (4)$$

In order to simplify the preceding differential equation, let us pose:

$$A_1 = \frac{D_e D_x}{bn^*} \quad (5)$$

$$A_2 = \left[\frac{D_e}{bn^*}\left(bn^* + \frac{1}{\tau_x}\right) + \frac{D_x}{bn^*}\left(bN_A + \frac{1}{\tau_e}\right)\right] \quad (6)$$

$$A_3 = \left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*} + \frac{1}{bn^*\tau_e}\right) + \frac{1}{\tau_e}\right] \quad (7)$$

$$Z = \left[G_{ox} + G_{oe} + \frac{G_{oe}}{bn^*}\left(\frac{1}{\tau_x} - D_x\alpha^2\right)\right] \quad (8)$$

The new form of differential equation becomes:

$$A_1\frac{d^4\Delta n_e}{dx^4} - A_2\frac{d^2\Delta n_e}{dx^2} + A_3\Delta n_e = Z\exp(-\alpha x) \quad (9)$$

The Laplace transform of equation (9) gives:

$$\Delta N_{eL}(P) = \frac{A_2\sum_0^1 P^{(K)}\Delta n_e^{(1-K)} - A_1\sum_0^3 P^{(3-K)}\Delta n_e^{(K)}}{A_1P^4 - A_2P^2 + A_3} + \frac{Z}{(P+\alpha)\left(A_1P^4 - A_2P^2 + A_3\right)} \quad (10)$$

- **CELL IN DARK**

The first expression $\Delta N_e(P)$ permit to determine the solution of differential equation without second member corresponding to excess minority carriers density in silicon base in dark.

$$\Delta N_e(P) = \frac{A_2\sum_0^1 P^{(K)}\Delta n_e^{(1-K)} - A_1\sum_0^3 P^{(3-K)}\Delta n_e^{(K)}}{A_1P^4 - A_2P^2 + A_3} \quad (11)$$

$\Delta n^{(k)}_{oe}$ is the kth-derivatives of excess minority carriers density in base at the junction.

The differential equation (11) in simplified form becomes:

International Journal of Engineering Research ISSN:2319-6890(online),2347-5013(print)
Volume No.6, Issue No.1, pp :37-42 1 Jan.2017

$$\Delta N_e(P)=\frac{-B_1P^3-B_2P^2+B_3P+B_4}{A_1P^4-A_2P^2+A_3} \quad (12)$$

With

$$\begin{aligned}B_1&=A_1\Delta n_{oe}\\B_2&=A_1\Delta n^{(1)}{}_{oe}\\B_3&=A_2\Delta n_{oe}-A_1\Delta n^{(2)}{}_{oe}\\B_4&=A_2\Delta n^{(1)}{}_{oe}-A_1\Delta n^{(3)}{}_{oe}\end{aligned} \quad (13)$$

Decompose the equality (12) on the sum of two terms. By doing so, determine the values α_1, α_2, α_3, α_4, α_5, α_6 and α_7 such as :

$$\frac{\alpha_1P+\alpha_2}{\alpha_3P^2+\alpha_4}+\frac{\alpha_5P+\alpha_6}{\alpha_3P^2+\alpha_7}=\frac{-B_1P^3-B_2P^2+B_3P+B_4}{A_1P^4-A_2P^2+A_3} \quad (14)$$

By identification, we obtain the following system:

$$\begin{cases}\alpha_3{}^2=A_1\\\alpha_3\alpha_7+\alpha_3\alpha_4=-A_2\\\alpha_4\alpha_7=A_3\\\alpha_1\alpha_3+\alpha_3\alpha_5=-B_1\\\alpha_2\alpha_3+\alpha_3\alpha_6=-B_2\\\alpha_1\alpha_7+\alpha_5\alpha_4=B_3\\\alpha_2\alpha_7+\alpha_4\alpha_6=B_4\end{cases} \quad (15)$$

We obtain the α_i coefficients (see annex).

$$\Delta N_e(P)=\frac{\alpha_1P+\alpha_2}{\alpha_3P^2+\alpha_4}+\frac{\alpha_5P+\alpha_6}{\alpha_3P^2+\alpha_7} \quad (16)$$

Let us pose $\beta_1=\sqrt{-\frac{\alpha_4}{\alpha_3}}$ and $\beta_2=\sqrt{-\frac{\alpha_7}{\alpha_3}}$ (17)

Boundary conditions:

$$\Delta n_e(0)=\frac{n_i^2}{N_A}\left[\exp\left(\frac{qV_a}{kT}\right)-1\right] \quad (18)$$

$$D_e\frac{d\Delta n_e(x)}{dx}\Big|_{x=0}=Sf\ \Delta n_e(0) \quad (19)$$

$$D_e\frac{d^2\Delta n_e}{dx^2}\Big|_{x=0}=\Delta n_e(0)\left[\frac{1}{\tau_e}+bN_A\right] \quad (20)$$

$$D_e\frac{d^3\Delta n_e(x)}{dx^3}\Big|_{x=0}=\frac{Sf\ \Delta n_e(0)}{D_e}\left[\frac{1}{\tau_e}+bN_A\right] \quad (21)$$

$$\Delta n_e(+\infty)=0 \quad (22)$$

When applying the reverse Laplace transform to equation (16), the expression of the excess electrons density in the base depending to the base depth and different cell parameters such as the doping level, the binding coefficient between electrons and excitons, etc...,becomes:

$$\Delta n_e(x)=\left[\frac{\alpha_1}{\alpha_3}\right]\cosh(\beta_1x)+\left[\frac{\alpha_2}{\alpha_3\beta_1}\right]\sinh(\beta_1x)+\left[\frac{\alpha_5}{\alpha_3}\right]\cosh(\beta_2x)+\left[\frac{\alpha_6}{\alpha_3\beta_2}\right]\sinh(\beta_2x)$$

(23)

- **CELL IN ILLUMINATION**

The second term permits to determine the particular solution of the equation with the second member. It corresponds to the Laplace transformation of the photogenerated excess minority carriers density.

$$\Delta N_{e\alpha}(P)=\frac{Z}{(P+\alpha)\left(A_1P^4-A_2P^2+A_3\right)} \quad (24)$$

Determine the reel number $a_i$such as:

$$\Delta N_{e\alpha}(P)=\frac{Z}{(P+\alpha)\left(A_1P^4-A_2P^2+A_3\right)}=\frac{a_1}{P+\alpha}+\frac{a_2P+a_4}{a_3P^2+a_5}+\frac{a_6P+a_7}{a_3P^2+a_8}$$

(25)

By identification, we obtain two systems:

$$\begin{cases}a_3^2=A_1\\a_5a_8=A_3\\a_3(a_5+a_8)=-A_2\end{cases}$$

$$\begin{pmatrix}A_3&0&\theta_1&0&\theta_2\\0&\theta_1&a_8&\theta_2&a_5\\a_3&1&0&1&0\\0&\theta_3&a_3&\theta_3&a_3\\\theta_4+\theta_5&a_8&\theta_3&a_5&\theta_3\end{pmatrix}\begin{pmatrix}a_1\\a_2\\a_4\\a_6\\a_7\end{pmatrix}=\begin{pmatrix}Z\\0\\0\\0\\0\end{pmatrix} \quad (26)$$

We can solve this system by Maxwell method.
Calculation of the matrix determinant:

$$M=\begin{pmatrix}A_3&0&\theta_1&0&\theta_2\\0&\theta_1&a_8&\theta_2&a_5\\a_3&1&0&1&0\\0&\theta_3&a_3&\theta_3&a_3\\\theta_4+\theta_5&a_8&\theta_3&a_5&\theta_3\end{pmatrix} \quad (27)$$

Boundary conditions:

$$\Delta n_e(0)=0 \quad (28)$$

$$\Delta n_e^{(1)}(x)\Big|_{x=0}=0 \quad (29)$$

$$D_e\frac{d^2\Delta n_e(x)}{dx^2}\Big|_{x=0}=-G_{oe} \quad (30)$$

$$D_e\frac{d^3\Delta n_e(x)}{dx^3}\Big|_{x=0}=+\alpha G_{oe} \quad (31)$$

$$\Delta n_e(+\infty)=0 \quad (32)$$

The resolution of this equation system (20) (see annex) is allow to us to determine the particular solution of the differential equation.

$$\Delta n_{e\alpha} = \frac{a_2}{\alpha_3}\cosh(\beta_1 x) + \frac{a_4}{\alpha_3\beta_1}\sinh(\beta_1 x) + \frac{a_6}{\alpha_3}\cosh(\beta_2 x) + \frac{a_7}{\alpha_3\beta_2}\sinh(\beta_2 x) + a_1\exp(-\alpha x) \quad (33)$$

The general expression of the excess minority carriers density is given:

$$\Delta n_{eL} = \Delta n_e + \Delta n_{e\alpha} \quad (34)$$

$$\Delta n_{epL} = \left[\frac{\alpha_1 + a_2}{\alpha_3}\right]\cosh(\beta_1 x) + \left[\frac{\alpha_2 + a_4}{\alpha_3\beta_1}\right]\sinh(\beta_1 x) + \left[\frac{\alpha_5 + a_6}{\alpha_3}\right]\cosh(\beta_2 x) + \left[\frac{\alpha_6 + a_7}{\alpha_3\beta_2}\right]\sinh(\beta_2 x) + a_1\exp(-\alpha x) \quad (35)$$

A. EXPRESSION OF EXCESS EXCITONS DENSITY

The excitons density in illuminated solar cell base is equal to the sum of the thermogenerated (expression without second member) and photogeneratedexcitons density (particular solution). We can deduce from the equation (3):

We obtain the following expression:

$$\begin{aligned}\Delta n_{xL} = &\left[\left(\frac{\alpha_1 + a_2}{\alpha_3 bn^*}\right)\left(bN_A + \frac{1}{\tau_e}\right) - \frac{D_e(\beta_1)^2(\alpha_1 + a_2)}{bn^*\alpha_3}\right]\cosh(\beta_1 x)\\ &+ \left[\left(\frac{\alpha_2 + a_4}{\alpha_3\beta_1 bn^*}\right)\left(bN_A + \frac{1}{\tau_e}\right) - \frac{D_e\beta_1(\alpha_2 + a_4)}{bn^*\alpha_3}\right]\sinh(\beta_1 x)\\ &+ \left[\left(\frac{\alpha_5 + a_6}{\alpha_3 bn^*}\right)\left(bN_A + \frac{1}{\tau_e}\right) - \frac{D_e(\beta_2)^2(\alpha_5 + a_6)}{bn^*\alpha_3}\right]\cosh(\beta_2 x)\\ &+ \left[\left(\frac{\alpha_6 + a_7}{\alpha_3\beta_2 bn^*}\right)\left(bN_A + \frac{1}{\tau_e}\right) - \frac{D_e\beta_2(\alpha_6 + a_7)}{bn^*\alpha_3}\right]\sinh(\beta_2 x)\\ &+ \left[\frac{a_1}{bn^*}\left(bN_A + \frac{1}{\tau_e}\right) - \frac{\alpha^2 a_1 D_e}{bn^*} - \frac{G_{oe}}{bn^*}\right]\exp(-\alpha x) \quad (36)\end{aligned}$$

CONCLUSION

This solving method allows getting a new expression of excess minority carriers' density and that of excitons in the base. The expressions of excess minority carriers density will permit to do a comparative study of some profiles obtain from different methods. These comparative studies will permit to estimate the second and third derivated values of excess minority carriers density at the junction.

Studies more deepened of the variations of the current density in function to these parameters will permit to know these physical meanings.

REFERENCE

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NOMENCLATURE

symbols	*Name and unit*
Δn	Excess minority carrier density, cm^{-3}
Δn_x	Exciton density, cm^{-3}
b	Binding coefficient, $cm^3.s^{-1}$
G_{eh0}	direct generation rate of carrier pairs, $cm^{-3}.s^{-1}$
G_{x0}	Exciton generation rate at the semiconductor surface, $cm^{-3}.s^{-1}$
Δn_{oe}	Excess minority carriers density at the junction, cm^{-3}
Δn_{xp}	Excess excitons density at the junction, cm^{-3}
x	the base depth, cm
N_A	Doping level, cm^{-3}
D_e	Diffusion coefficient for electron, $cm^2.s^{-1}$
D_x	Diffusion coefficient for excitons, $cm^2.s^{-1}$
T_e	Electrons lifetime, s
T_x	Excitons lifetime, s
H	Base thickness, cm
$n*$	Equilibrium constant, cm^{-3}
α	Absorption coefficient, cm^{-1}

ANNEX

After calculation, we obtain the α_i coefficients:

$$\alpha_3 = \sqrt{\frac{D_e D_x}{bn^*}} \quad (A\text{-}1)$$

$$\alpha_4 = -\frac{1}{2}\left[\frac{\frac{D_e}{bn^*}\left(bn^* + \frac{1}{\tau_x}\right) + \frac{D_x}{bn^*}\left(bN_A + \frac{1}{\tau_e}\right)}{\sqrt{\frac{D_e D_x}{bn^*}}}\right]\left[1 + \sqrt{1 - \frac{4\frac{D_e D_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*} + \frac{1}{bn\tau_e}\right) + \frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^* + \frac{1}{\tau_x}\right) + \frac{D_x}{bn^*}\left(bN_A + \frac{1}{\tau_e}\right)\right]^2}}\right] \quad (A\text{-}2)$$

$$\alpha_7=\frac{1}{2}\left[\frac{\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)}{\sqrt{\frac{D_eD_x}{bn^*}}}\right]\left[-1+\sqrt{1-\frac{4.\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}\right] \quad \text{(A-3)}$$

$$\alpha_6=\frac{\frac{\Delta n'''_{0e}\left(\frac{D_eD_x}{bn^*}\right)^{\frac{3}{2}}}{\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)}-\frac{1}{2}\Delta n'_{oe}\sqrt{\frac{D_eD_x}{bn^*}}\left[1+\sqrt{1-\frac{4\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}\right]}{\sqrt{1-\frac{4\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}} \quad \text{(A-4)}$$

$$\alpha_2=\frac{\frac{-\Delta n'''_{0e}\left(\frac{D_eD_x}{bn^*}\right)^{\frac{3}{2}}}{\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)}+\frac{1}{2}\Delta n'_{oe}\sqrt{\frac{D_eD_x}{bn^*}}\left[1-\sqrt{1-\frac{4.\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}\right]}{\sqrt{1-\frac{4\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}} \quad \text{(A-5)}$$

$$\alpha_5=\frac{\frac{\Delta n''_{0e}\left(\frac{D_eD_x}{bn^*}\right)^{\frac{3}{2}}}{\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)}-\frac{1}{2}\Delta n_{0e}\sqrt{\frac{D_eD_x}{bn^*}}\left[1+\sqrt{1-\frac{4.\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}\right]}{\sqrt{1-\frac{4\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}} \quad \text{(A-6)}$$

$$\alpha_1=\frac{\frac{\Delta n''_{0e}\left(\frac{D_eD_x}{bn^*}\right)^{\frac{3}{2}}}{\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)}-\frac{1}{2}\Delta n_{0e}\sqrt{\frac{D_eD_x}{bn^*}}\left[1+\sqrt{1-\frac{4.\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}\right]}{\sqrt{1-\frac{4\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}} \quad \text{(A-7)}$$

$$\beta_2=\sqrt{\frac{1}{2}\left[\frac{\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)}{\frac{D_eD_x}{bn^*}}\right]\left[1-\sqrt{1-\frac{4\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}\right]} \quad \text{(A-8)}$$

$$\beta_2=\sqrt{\frac{1}{2}\left[\frac{\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)}{\frac{D_eD_x}{bn^*}}\right]\left[1+\sqrt{1-\frac{4\frac{D_eD_x}{bn^*}\left[\frac{1}{\tau_x}\left(\frac{N_A}{n^*}+\frac{1}{bn\tau_e}\right)+\frac{1}{\tau_e}\right]}{\left[\frac{D_e}{bn^*}\left(bn^*+\frac{1}{\tau_x}\right)+\frac{D_x}{bn^*}\left(bN_A+\frac{1}{\tau_e}\right)\right]^2}}\right]} \quad \text{(A-9)}$$

Determination of $a_i$coefficients

$$\Delta N_{e\alpha}(P)=\frac{Z}{(P+\alpha)\left(A_1P^4-A_2P^2+A_3\right)} \quad \text{(A-10)}$$

For applying the inverse Laplace transformation, we determine a_i reel values such as:

$$\Delta N_{e\alpha}(P)=\frac{Z}{(P+\alpha)\left(A_1P^4-A_2P^2+A_3\right)}=\frac{a_1}{P+\alpha}+\frac{a_2P+a_4}{a_3P^2+a_5}+\frac{a_6P+a_7}{a_3P^2+a_8} \quad \text{(A-11)}$$

By identification, we obtain the following systems:

$$\begin{cases} a_3^2=A_1 \\ a_5a_8=A_3 \\ a_3\left(a_5+a_8\right)=-A_2 \end{cases} \quad \text{(A-12)}$$

$$\begin{pmatrix} A_3 & 0 & \theta_1 & 0 & \theta_2 \\ 0 & \theta_1 & a_8 & \theta_2 & a_5 \\ a_3 & 1 & 0 & 1 & 0 \\ 0 & \theta_3 & a_3 & \theta_3 & a_3 \\ \theta_4+\theta_5 & a_8 & \theta_3 & a_5 & \theta_3 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_4 \\ a_6 \\ a_7 \end{pmatrix}=\begin{pmatrix} Z \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{(A-13)}$$

The resolution of the system (A-13) gives:

$a_3=\alpha_3$ $a_4=\alpha_4$ $a_7=\alpha_7$ (A-14)

The second system of equation obtained by identification is:

$$\begin{cases} A_3a_1+\theta_1a_4+\theta_2a_7=Z \\ \theta_1a_2+a_8a_4+\theta_2a_6+a_5a_7=0 \\ a_3a_1+a_2+a_6=0 \\ \theta_3a_2+a_3a_4+\theta_3a_6+a_3a_7=0 \\ \left(\theta_4+\theta_5\right)a_1+a_8a_2+\theta_3a_4+a_5a_6+\theta_3a_7=0 \end{cases} \quad \text{(A-15)}$$

with

$\theta_1=\alpha\ a_8$ $\theta_3=\alpha\ a_3$ $\theta_2=\alpha\ a_5$ $\theta_4=a_3a_8$ $\theta_5=a_3a_5$ (A-16)

$\Delta_M=\det(M)$ (A-17)

$$\Delta_M=\left(a_3a_5-\theta_3^2\right)\left[A_3a_8+a_3\left(\theta_1^2-\theta_1\theta_2\right)\right]+\left(a_5-a_8\right)\left[A_3a_3a_8+a_3a_8\theta_2\theta_3-a_3a_5\theta_1\theta_3\right] \\ +a_3\left(\theta_3^2-a_3a_8\right)\left(\theta_1\theta_2-\theta_2^2\right)-a_3\left(\theta_4+\theta_5\right)\left(\theta_1-\theta_2\right)^2+A_3a_3\left(\theta_3a_8-a_5^2\right) \quad \text{(A-18)}$$

$$\Delta a_1 = \det\begin{pmatrix} Z & 0 & \theta_1 & 0 & \theta_2 \\ 0 & \theta_1 & a_8 & \theta_2 & a_5 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & \theta_3 & a_3 & \theta_3 & a_3 \\ 0 & a_8 & \theta_3 & a_5 & \theta_3 \end{pmatrix} \quad \Delta a_1 = Z\begin{bmatrix} a_8\left(a_3a_5-\theta_3^2\right)+a_3\left(\theta_2\theta_3-a_5^2\right) \\ +\theta_3\left(\theta_3a_5-a_3\theta_2\right)+A_3\theta_3\left(a_3a_8-\theta_3a_5\right) \\ +A_3a_8\left(a_3a_5-a_8a_3\right) \end{bmatrix} \quad a_1 = \frac{\Delta a_1}{\Delta_M} \quad \text{(A-21)}$$

$$\Delta a_2 = \det\begin{pmatrix} A_3 & Z & \theta_1 & 0 & \theta_2 \\ 0 & 0 & a_8 & \theta_2 & a_5 \\ a_3 & 0 & 0 & 1 & 0 \\ 0 & 0 & a_3 & \theta_3 & a_3 \\ \theta_4+\theta_5 & 0 & \theta_3 & a_5 & \theta_3 \end{pmatrix} \quad \Delta a_2 = Za_3\begin{Bmatrix} a_8\left(\theta_3^2-a_3a_5\right)-a_3\left(\theta_2\theta_3-a_5^2\right) \\ +\theta_3\left(\theta_2a_3-\theta_3a_5\right) \\ +\left(\theta_4+\theta_5\right)\left(a_8-a_5\right) \end{Bmatrix} \quad a_2 = \frac{\Delta a_2}{\Delta_M} \quad \text{(A-24)}$$

$$\Delta a_4 = \det\begin{pmatrix} A_3 & 0 & Z_1 & 0 & \theta_2 \\ 0 & \theta_1 & 0 & \theta_2 & a_5 \\ a_3 & 1 & 0 & 1 & 0 \\ 0 & \theta_3 & 0 & \theta_3 & a_3 \\ \theta_4+\theta_5 & a_8 & 0 & a_5 & \theta_3 \end{pmatrix} \quad \Delta a_4 = a_3Z\begin{bmatrix} \theta_1\left(a_3a_5-a_3\theta_3\right) \\ +\theta_3\left(\theta_2a_3-a_5^2\right) \\ +a_8\left(\theta_3a_5-a_3\theta_2\right) \\ +\left(\theta_4+\theta_5\right)\left(\theta_2-\theta_1\right) \end{bmatrix} \quad a_4 = \frac{\Delta a_4}{\Delta_M} \quad \text{(A-27)}$$

$$\Delta a_6 = \det\begin{pmatrix} A_3 & 0 & \theta_1 & Z & \theta_2 \\ 0 & \theta_1 & a_8 & 0 & a_5 \\ a_3 & 1 & 0 & 0 & 0 \\ 0 & \theta_3 & a_3 & 0 & a_3 \\ \theta_4+\theta_5 & a_8 & \theta_3 & 0 & \theta_3 \end{pmatrix} \quad \Delta a_6 = Z\left(a_5-a_8\right)\left[\theta_3^2-a_3a_8+a_3\left(\theta_4+\theta_5\right)\right] \quad a_6 = \frac{\Delta a_6}{\Delta_M} \quad \text{(A-29)}$$

$$\Delta a_7 = \det\begin{pmatrix} A_3 & 0 & \theta_1 & 0 & Z \\ 0 & \theta_1 & a_8 & \theta_2 & 0 \\ a_3 & 1 & 0 & 1 & 0 \\ 0 & \theta_3 & a_3 & \theta_3 & 0 \\ \theta_4+\theta_5 & a_8 & \theta_3 & a_5 & 0 \end{pmatrix} \quad \Delta a_7 = a_3Z\begin{bmatrix} \theta_1\left(\theta_3^2-a_3a_5\right)+\theta_3\left(a_8a_5-\theta_2\theta_3\right) \\ +a_8\left(a_3\theta_2-a_8\theta_3\right) \\ +\left(\theta_4+\theta_5\right)\left(\theta_1-\theta_2\right) \end{bmatrix} \quad a_7 = \frac{\Delta a_7}{\Delta_M} \quad \text{(A-32)}$$